ARKIV FÖR FYSIK Band 2 nr 10

Communicated 23 November 1949 by G. Borelius and H. Alfvén

A study of the effect of tension on the electrical resistance of alloys. I.

By J. O. Linde, Axel von Heijne and E. von Sabsay

With 6 figures in the text

Introduction

The present work, of which this paper constitutes the first part, covers a general study of the dependence of strain on the electrical resistance of alloys whose main constituent is copper, silver or gold with the addition of other elements. The practical aim is to develop alloys suitable for wire strain gauges. For this reason temperature-coefficients of resistance and thermoelectric power against copper have been determined for some of the alloys in the range 0°—100° C. These will be reported in a later paper.

At the end of the paper a fairly complete list of literature is given of papers covering the resistance-strain effect, practical and theoretical works on wire resistance gauges, and papers which have been cited in the text.

1. Preparation of samples

The alloys were prepared by melting together the carefully weighed components in evacuated silica tubes. After rolling and homogenizing, the rods were drawn through cemented carbide dies to wires about 0.2 mm in diameter. The measurements were normally made on the wires in the hard-drawn state but in some cases also after annealing at 100° or 200° C.

2. Apparatus for the measurement of resistance and strain

A Wheatstone bridge was used for the resistance measurements. In Fig. 1 R_x is the sample wire and R_2 a wire of the same alloy and dimensions but unstrained. A third wire of copper could be switched in to serve as a resistance thermometer. The other resistances in the bridge were of manganin. In order to keep the temperature constant the wires were contained in a copper tube wrapped round with glass wool. Temperature measurements were made at intervals, but it was found that the balanced bridge enabled the influence of temperature variations to be kept within the experimental error. The switch

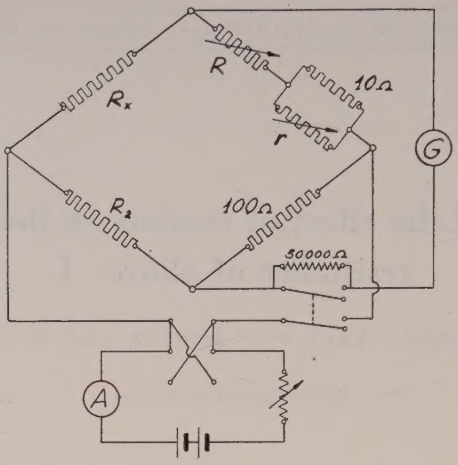

Fig. 1.

operating the main current and the galvanometer was of the usual construction allowing the galvanometer circuit to be closed somewhat later than the bridge circuit. The current was maintained at about 6 mA. Errors due to thermo-e.m.fs. were eliminated by reversing the bridge current.

The resistance R was chosen so that resistance r (about 1000 ohms, variable in four decades) could be fully utilized. Using the symbols of Fig. 1, when no current flows through the galvanometer, then

$$\frac{R_x}{R_2} = \frac{R + \dfrac{10\,r}{10 + r}}{100} \tag{1}$$

or, by putting

$$b = R + \frac{10\,r}{10 + r}, \qquad R_x = R_2 \frac{b}{100}. \tag{1 a}$$

The absolute resistance of R_x was obtained by exchanging R_2 for a standard resistance and measuring R_x unstrained.

In order to measure the strain simultaneously with the resistance, the apparatus shown in Fig. 2 was devised. A is a stationary tube of insulating material (ebonite) in which a copper rod A_1 can be fastened. The sample and comparison wires were soldered at one end to A_1. The other end of the sample wire was soldered to a similar copper rod in an ebonite tube B, freely suspended by two spun silk threads. The threads passed round two grooves and their upper ends were fastened to four screws S which enabled the movable rod to be positioned vertically. The comparison wire was soldered to another copper rod in a stationary tube C. The sample wire was stressed by weights placed on a balance pan attached to B by means of a silk thread carried on

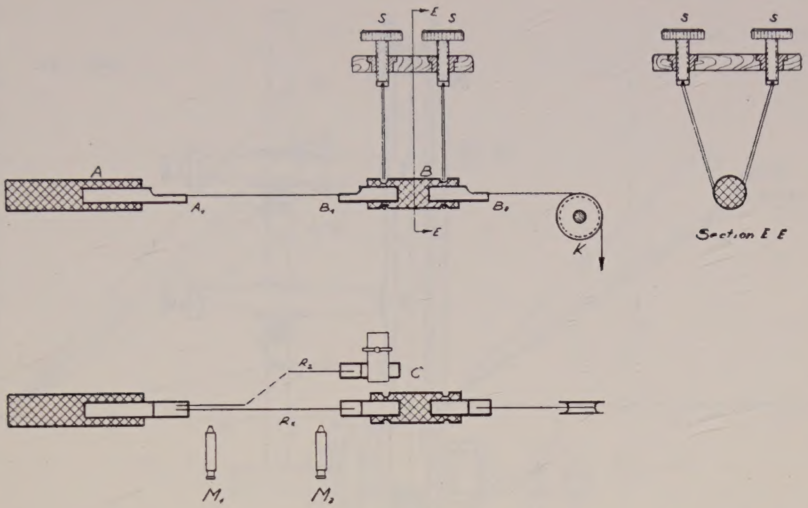

Fig. 2.

a ball-bearing roller K (SKF 291817). The pan weighed about 60 grams. The change in length caused by a change in load was determined with two microscopes M_1 and M_2 fitted with eye-piece micrometers, which were mounted near the ends of the sample wire and sighted on it through holes in the insulating copper tube. An index was obtained on the wire by painting a water-colour mark with a fine brush. Each scale degree on the eye-piece micrometers corresponded to 7.8×10^{-4} mm.

Finally the apparatus shown in Fig. 3 was constructed to permit accurate determination of the Young's modulus of the alloys. Here the wires were loaded vertically since there is liable to be less error in the stressing force than in the horizontal arrangement; and because the microscopic determination of the strain can be made more accurately, since the position of the wire is less variable and can be seen distinctly in the microscope. The stress-strain measurements were made in this apparatus, while resistance-stress measurements and sometimes also the resistance-strain measurements were made in the horizontal apparatus (Fig. 2).

3. Results

Measurements have been made on the copper alloys Cu-Si, Cu-Ni, and the technical alloy manganin; on the silver alloys Ag-Sn; and on the pure metals copper and silver. In all cases the resistance was found to be a linear function of the stress within the experimental error (Fig. 4 a and 4 b). Table 1 shows for all tested samples the resistivity at 20° C (ϱ_{20}), the resistance-stress coefficient $\left(a = \frac{1}{R}\frac{dR}{d\sigma}\right)$ and the corresponding resistivity-stress coefficient $\left(a_\varrho = \frac{1}{\varrho}\frac{d\varrho}{d\sigma}\right)$, Young's modulus E, and the dimensionless strain-gauge factor g. The latter is defined by

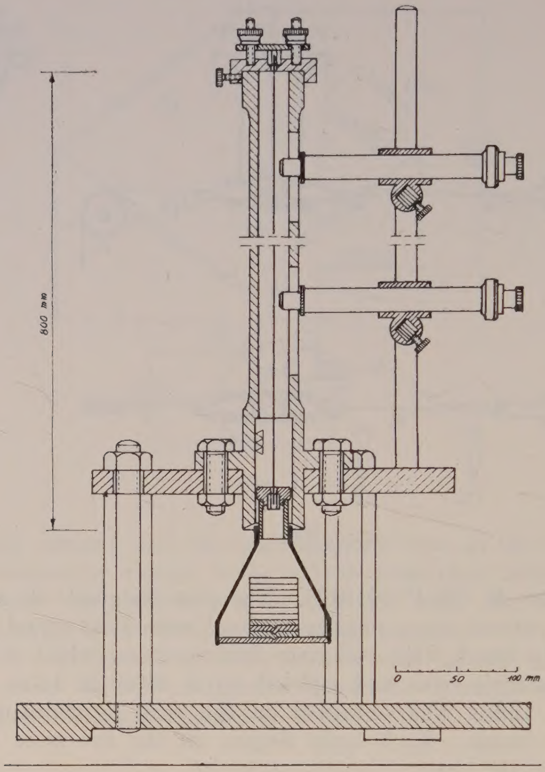

Fig. 3.

$$g = \frac{\Delta R}{R} : \frac{\Delta l}{l}$$

where $\frac{\Delta R}{R}$ is the relative change in resistance corresponding to the strain $\frac{\Delta l}{l}$. It can easily be seen that

$$g = aE\,.$$

The table shows that the strain-gauge factor g decreases continuously with the concentration c of additional element in all the alloy systems studied. Young's modulus E appears usually to decrease with increasing c in alloys with b-elements (Si and Sn) and to increase with c in alloys with Ni. However, the variation of E with concentration shows some irregularities which appear to be larger than the experimental errors and which are probably due to a certain preferred crystal orientation in the cold-drawn wires (texture). These irregularities can be caused by the degree of orientation varying from sample to sample. The corresponding irregularities can be seen in the values of the resistance-strain coefficient, a (Table 1).

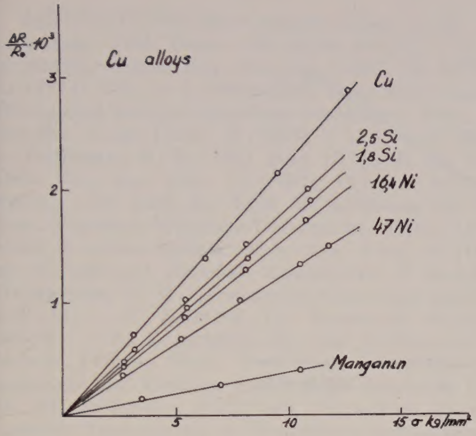

Fig. 4 a.

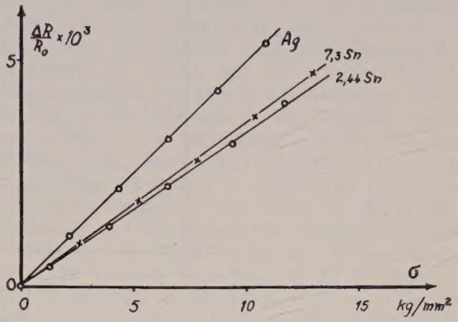

Fig. 4 b.

Table 1

(The figures before the chemical symbols indicate the concentration *c* of the solutes in atomic percentage. The specimens are all cold-worked.)

Specimen	$\varrho_{20} \times 10^6$ ohm cm (20° C)	$a \times 10^6$ kg^{-1} cm^2	$E \times 10^{-6}$ kg cm^{-2}	g	$a_\varrho \times 10^6$ ohm cm^3 kg^{-1}
Cu alloys					
Cu pure	1.7	2.21	1.25	2.76	0.85
0.65 Si	4.2	1.79	1.35	2.42	0.53
1.8 »	8.9	1.75	1.24	2.18	0.38
2.5 »	11.3	1.85	1.13	2.09	0.34
16.4 Ni	18.9	1.57	1.32	2.08	0.31
41.9 »	51.0	1.27	1.62	2.06	0.25
47.0 » (advance) .	55.4	1.26	1.63	2.05	0.24
Manganin	43.0	0.41	1.25	0.51	− 0.82
Ag alloys					
Ag pure	1.6	4.8	0.70	3.36	2.3
2.44 Sn	11.0	3.45	0.70	2.42	0.9
4.86 »	20.5	3.41	0.67	2.29	0.8
7.30 »	29.0	3.62	0.62	2.24	0.8

The tabulated values of the resistivity-strain coefficient, a_ϱ have been computed from a, E, and μ by the easily deduced equation

$$a_\varrho = a - \frac{1 + 2\mu}{E}$$

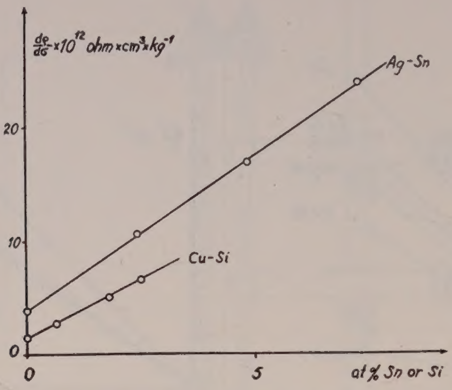

Fig. 5.

where μ is Poisson's number, expressing the ratio between transverse contraction and strain in one-dimensional elastic deformation. For those alloys where Poisson's number was not known, the value for the corresponding pure metal (Cu or Ag) was used. In low-concentration alloys the error due to this approximation is probably negligible.

Finally $\frac{d\varrho}{d\sigma}$ has been calculated by multiplying a_ϱ by the corresponding ϱ (at zero strain). In the alloy series studied, this quantity varies linearly with the concentration c or resistivity ϱ, as can be seen in Fig. 5, where $\frac{d\varrho}{d\sigma}$ is plotted as a function of c. The regularity is analogous to that found in the dependence of pressure on the resistivity of alloys (34).

Added in proof. In Fig. 5 the ordinate values must be corrected as regards Cu-Si. For correction the figure 10 on the axis is to be changed to 6 for these alloys.

SUMMARY

The dependence of strain on resistance has been determined for some copper and silver alloys, and in connection with these measurements the alloys' Young's modulus has been measured. The resistance-strain coefficient decreases normally with increasing concentration in dilute alloys. It is found that $\frac{d\varrho}{d\sigma}$ varies approximately linearly with the alloys' concentration. Young's modulus increases with Ni concentration in the copper-nickel alloys but decreases with concentration in the other alloy systems (Cu-Si and Ag-Sn).

Acknowledgment. The investigation has been supported by a grant from the government. The work is being continued.

Physical Department, Kungl. Tekniska Högskolan, Stockholm, November 1949.

LITERATURE: *The resistance tension effect:* 1. **Thomson, W.**: The electrodynamical qualities of metals. Phil. Trans. 146 (1856) 649/72. — 2. **Thomlinson, H.**: On the physical properties of metals. Trans. Roy. Soc. 174 (1883) 39, 1/172. — 3. **Chwolson**: J. Russ. Phys. Chem. Ges. 15 (1881), 153. — 4. **Mousson**: Neue Schweizer Z. 14 (1885), 33. — 5. **Gray** and **Henderson**: Mechanical stress on electrical resistance. Proc. Roy. Soc. (Phil.) 53 (1893), 76. — 6. **Cantone**: Rendic. R. dei Lincei. 6 (1897) I, 175. — 7. **Smith**: Phys. Rev. 28 (1909), 107/21, 429/37. — 8. **Williams, W. E.**: Phil. Mag. 13 (1909), 13. 635/43. — 9. **Donaldson, J. A.** and **Wilson, R.**: Proc. Roy. Soc. Edin. 27 (1907), 16/20. — 10. **Zavattiero, E.**: Rend. Acc. Lincei. 20 (1920), I, 48/54. — 11. **Lax, E.**: Über die Änderung des Widerstandes in Drähten durch Dehnung. Phil. Diss. Friedrich-Wilhelms-Universität. Berlin (1919). — 12. **Bridgman, P. W.**: Tension coefficient of certain metals. Proc. Am. Acad. 57 (1922), 41/66. — 13. **Bridgman, P. W.**: Tension on thermal and electrical conductivity of metals. Proc. Am. Acad. 59 (1923), 117/137. — 14. **Bridgman, P. W.**: Tension on transverse and longitudinal resistance of metals. 60 (1924/25), 423/449. — 15. **Rolnick, H.**: Resistance of metals under tension. Phys. Rev. (2), 36 (1930), 506/12. — 16. **Lopukhin, E.**: Der Einfluss der Dehnung auf die elektrische Leitfähigkeit der Metalle. Techn. Phys. U. S. S. R. IV (1937), 1, 25/35. — 17. **Czerlinsky, E.**: Widerstandsänderung von Drahten durch Zug. Jahrbuch 1938 d. D. Luftfahrtforschung S. II 377/380. — 18. **Allen**: Tension coefficient. Phys. Rev. 49 (1936), 3, 248/253. — 19. **Seth, J. B.** and **Chetan Anand**: Veränderung in Widerstand beim Spannen von Ni-drahten. Phys. Z. 29 (1928), 951/2 and 30 (1929), 32. — 20. **Bedi, R. S.**: Widerstandsänderung bei langgespannten Ni-draht. Phys. Z. 31 (1930), 180/182. — 21. **McKeehan, L. W.**: Resistance of permalloy and nickel under magnetization and tension. Phys. Rev. (2), 36 (1930), 948/977. — 22. **Norie Yamanaka**: On the elasto-resistance change of iron and some iron-nickel alloys. Sci. Rep. Tohok Imp. Univ. 29 (1940) 36/68. — 23. **Bridgman, P. W.**: On the electrical conductivity of metals. Phys. Rev. 9 (1917), 269/289. — 24. **Bridgman, P. W.**: On the electrical conductivity of metals. Phys. Rev. 17 (1021), 161/194. — 25. **Hume-Rothery, W.**: The Metallic State, electrical properties and theories. Oxford. 1931. — 26. **Cioff, P. P.**: Method of supporting and stretching wire for measurement of elasto-resistance. J. Opt. Soc. and Rev. Sci. Instr. 9 (1924), 53/60. — *Measurements with strain gauges:* 27. **Davis** and **Carlson**: Electric strain-meter. Proc. A. S. T. M. 32 (1932) II, 793/801. — 28. **Carlson**: Strain-meter. Engineering Record 114 (1935), 696/7. — 29. **Clark, D. S.** and **Dätwyler, G.**: Stress-strain relations under impact loading. Proc. A. S. T. M. 38 (1938) II, 98/106. — 30. **de Forest, A. V.** and **Leaderman, H.**: Development of electrical strain gages. N. A. C. A. Tech. Note No. 744 (1939). — 31. **de Forest, A. V.**: Characteristics and aircraft applications of wire resistance strain gages. Instr. 15 (1942), 112/4, and 136/7. — 32. **Karlén, T.** och **Ljungström, T.**: Statisk och dynamisk spänningsmätning med resistiva trådtöjningsgivare. Tekn. T. 76 (1946) 681/90. — 33. **Redshaw, S. C.**: The electrical measurement of strain. J. Roy. Aeronnaut. Soc. 50 (1946), 568/612. — *Miscellaneous references:* 34. **Linde, J. O.**: Elektrische Widerstandseigenschaften der verdünnten Legierungen des Kupfers, Silbers und Goldes. Lund 1939; Arkiv f. mat., astr. o. fysik *36* A Nr 10, 1948. — 35. **Guillet, L. fils**: Contribution à l'étude du module d'élasticité des métaux et alliages. Rev. Metallurgie 39 (1939), 497/521.

Tryckt den 27 mars 1950

Uppsala 1950. Almqvist & Wiksells Boktryckeri AB